



Variable speed pump-turbines

Issues for hydraulic machinery

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Variable speed pump-turbines – Issues for hydraulic machinery

- **Machine and system curves**
- **Power governing in pump mode**
- **Turbine best-efficiency head**
- **Pump mode cavitation at low head**
- **Pump start stability at high head**
- **Turbine mode stability**



Machine curves

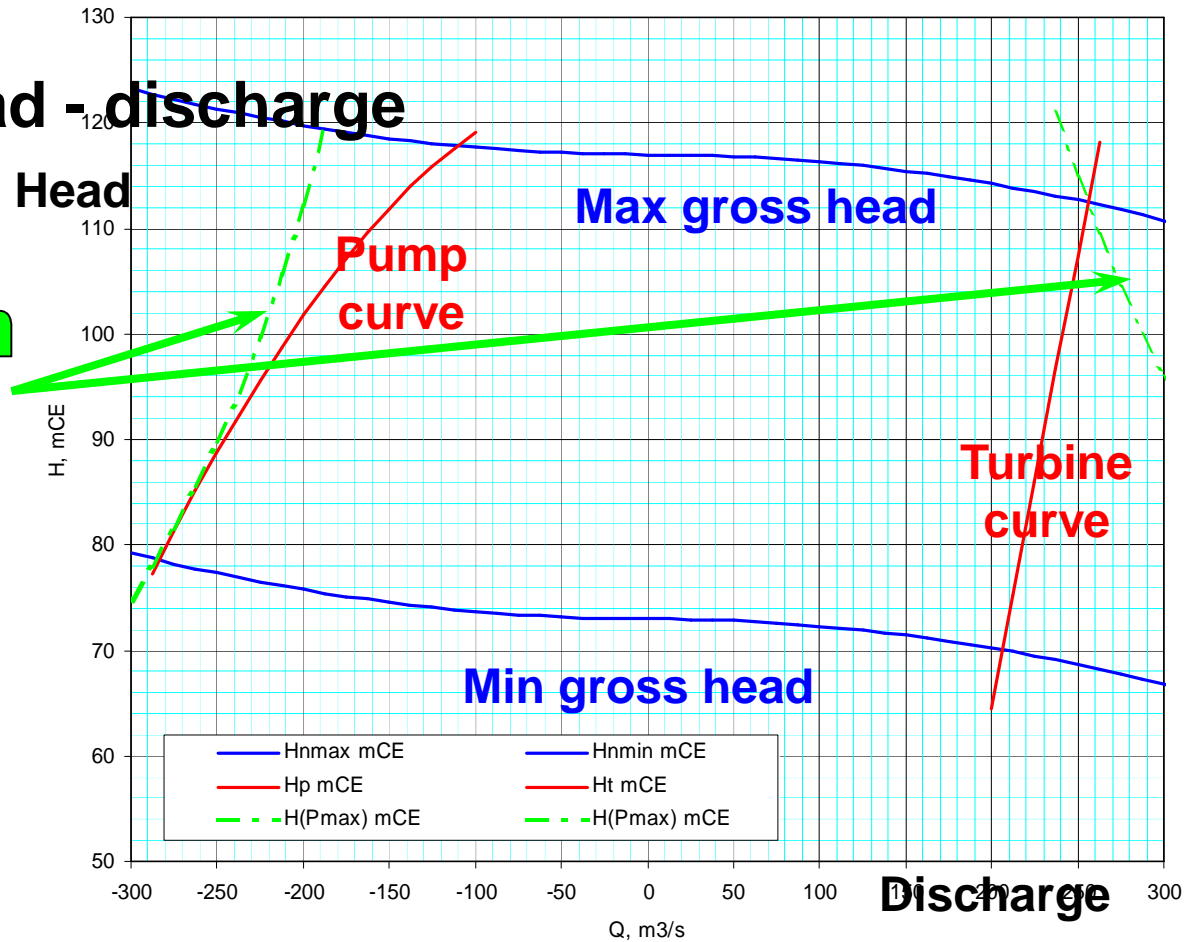
		Absolute	Coefficients	Suter	Factors
<p>ENTREE</p> <p>SORTIE</p> <p>Head Discharge</p> <p>V, H (V, H) $H = H(V)$ N donné N donné</p>	<p>Couple constant Vitesse de rotation constante</p>	$V = \pi R_0^2 \phi_{1a} \cdot N$ $H = \frac{R_0^2}{2g} \gamma_{1a} \cdot N^2$ $V = \phi_{1a} \sqrt{\frac{2N \cdot \pi^2 R_{1a}}{ \tau_{1a} }} \cdot \sqrt{ H }$ $H = \frac{\gamma_{1a}}{\tau_{1a}} \frac{1}{2N \cdot \mu \cdot \pi^2 R_{1a}} \cdot T$	$V = \pi R_0^2 \phi_{1a} \cdot N$ $H = \frac{R_0^2}{2g} \gamma_{1a} \cdot N^2$ $V = \phi_{1a} \sqrt{\frac{2N \cdot \pi^2 R_{1a}}{ \tau_{1a} }} \cdot \sqrt{ H }$ $H = \frac{\gamma_{1a}}{\tau_{1a}} \frac{1}{2N \cdot \mu \cdot \pi^2 R_{1a}} \cdot T$	$V = \pi R_0^2 \frac{c_{m1}}{u_{11}} \cdot N$ $H = \frac{R_0^2}{2g} \frac{1}{u_{11}^2} \cdot N^2 \cdot \text{sgn}(H)$ $V = \sqrt{\frac{2 \pi R_{11}}{\mu}} \frac{c_{m1}}{\sqrt{ \tau_{11} }} \cdot \sqrt{ H }$ $H = \frac{1}{\mu \cdot \pi R_{11}^2} \frac{1}{g} \frac{1}{\tau_{11}} \cdot T$	$V = \pi R_0^2 \frac{c_{m1}}{u_{11}} \cdot N$ $H = \frac{R_0^2}{2g} \frac{1}{u_{11}^2} \cdot N^2 \cdot \text{sgn}(H)$ $V = \sqrt{\frac{2 \pi R_{11}}{\mu}} \frac{c_{m1}}{\sqrt{ \tau_{11} }} \cdot \sqrt{ H }$ $H = \frac{1}{\mu \cdot \pi R_{11}^2} \frac{1}{g} \frac{1}{\tau_{11}} \cdot T$
<p>Head Discharge</p> <p>$\phi_{1a}, \gamma_{1a}, \tau_{1a}$ (phi, gamma, tau) $\gamma_{1a} = \gamma_{1a}(\phi_{1a})$ $\tau_{1a} = \tau_{1a}(\phi_{1a})$</p>	$\gamma_{1a} = \frac{g \cdot H}{ R_{1a} \cdot N ^2 / 2}$ $\phi_{1a} = \frac{V / \pi R_{1a}}{R_{1a} \cdot N}$ $\tau_{1a} = T \frac{N (2N / \mu \cdot \pi^2 R_{1a}^2)}{ R_{1a} \cdot N ^3}$	<p>γ_{1a} en fonction de ϕ_{1a} τ_{1a} en fonction de ϕ_{1a}</p>	$\gamma_{1a} = \gamma_{1a0} W_{u1} \cdot W_{\tau1} [1 + \cotg^2 \theta]$ $\phi_{1a} = \phi_{1a0} \cotg \theta$ $\tau_{1a} = \tau_{1a0} W_{u1} \cdot W_{\tau1} [1 + \cotg^2 \theta]$	$\gamma_{1a} = \frac{R_{11}^2}{R_{1a}^2} \frac{1}{u_{11}} \cdot \text{sgn}(H)$ $\phi_{1a} = \frac{R_{11}^2}{R_{1a}^2} \frac{c_{m1}}{u_{11}}$ $\tau_{1a} = \frac{1}{2N \cdot \mu \cdot \pi^2} \frac{R_{11}^2}{R_{1a}^2} \frac{\tau_{11}}{u_{11}}$	$\gamma_{1a} = \frac{R_{11}^2}{R_{1a}^2} \frac{1}{u_{11}} \cdot \text{sgn}(H)$ $\phi_{1a} = \frac{R_{11}^2}{R_{1a}^2} \frac{c_{m1}}{u_{11}}$ $\tau_{1a} = \frac{1}{2N \cdot \mu \cdot \pi^2} \frac{R_{11}^2}{R_{1a}^2} \frac{\tau_{11}}{u_{11}}$
<p>Head Quadrant</p> <p>θ, W_u, W_τ (theta, Wu, Wtau) $W_u = W_u(\theta)$ $W_\tau = W_\tau(\theta)$</p>	$\theta = \arctg \frac{N/N_0}{V/V_0}$ $W_u = \sqrt{\frac{ H / H_0 }{(N/N_0)^2 + (V/V_0)^2}} \cdot \text{sgn}(H)$ $W_\tau = \sqrt{\frac{ \tau_{11} /\tau_{1a0}}{(N/N_0)^2 + (V/V_0)^2}} \cdot \text{sgn}(\tau_{1a})$	$\theta = \arctg \frac{\phi_{1a}}{\phi_{1a0}}$ $W_u = \sqrt{\frac{ \gamma_{1a} /\gamma_{1a0}}{1 + (\phi_{1a}/\phi_{1a0})^2}} \cdot \text{sgn}(\gamma_{1a})$ $W_\tau = \sqrt{\frac{ \tau_{1a} /\tau_{1a0}}{1 + (\phi_{1a}/\phi_{1a0})^2}} \cdot \text{sgn}(\tau_{1a})$	<p>W_{u1} en fonction de θ $W_{\tau1}$ en fonction de θ</p>	$\theta = \arctg \left(\frac{c_{m1} u_{11}}{u_{10}} \right)$ $W_u = \frac{1}{\sqrt{(u_{11}/u_{10})^2 + (c_{m1}/c_{m10})^2}} \cdot \text{sgn}(H)$ $W_\tau = \frac{ \tau_{11} /\tau_{1a0}}{\sqrt{(u_{11}/u_{10})^2 + (c_{m1}/c_{m10})^2}} \cdot \text{sgn}(\tau_{1a})$	$\theta = \arctg \left(\frac{c_{m1} u_{11}}{u_{10}} \right)$ $W_u = \frac{1}{\sqrt{(u_{11}/u_{10})^2 + (c_{m1}/c_{m10})^2}} \cdot \text{sgn}(H)$ $W_\tau = \frac{ \tau_{11} /\tau_{1a0}}{\sqrt{(u_{11}/u_{10})^2 + (c_{m1}/c_{m10})^2}} \cdot \text{sgn}(\tau_{1a})$
<p>Speed Discharge</p> <p>u_{11}, c_{m11}, s_{11} (u, cm, s) $c_{m11} = c_{m11}(u_{11})$ $s_{11} = s_{11}(u_{11})$</p>	$u_{11} = \frac{N R_{11}}{\sqrt{2 \cdot g \cdot H }}$ $c_{m11} = \frac{V}{\pi R_{11}^2 \sqrt{2 \cdot g \cdot H }}$ $s_{11} = \frac{T}{\mu \cdot \pi R_{11}^2 \cdot g \cdot H}$	$u_{11} = \frac{R_{11}}{R_{1a}} \frac{1}{\sqrt{ \gamma_{1a} }} \cdot \text{sgn}(H)$ $c_{m11} = \frac{R_{11}^2}{R_{1a}^2} \frac{\phi_{1a}}{\sqrt{ \gamma_{1a} }}$ $s_{11} = 2N \cdot \pi^2 \frac{R_{11}^2}{R_{1a}^2} \frac{\tau_{1a}}{\gamma_{1a}}$	$u_{11} = u_{110} \frac{\sin \theta}{ W_{u1} }$ $c_{m11} = c_{m10} \frac{\cos \theta}{ W_{u1} }$ $s_{11} = s_{110} \frac{ W_{\tau1} \cdot W_{\tau1} }{W_{u1}^2}$	<p>s_{11} en fonction de u_{11} c_{m11} en fonction de u_{11}</p>	$u_{11} = u_{110} \frac{\sin \theta}{ W_{u1} }$ $c_{m11} = c_{m10} \frac{\cos \theta}{ W_{u1} }$ $s_{11} = s_{110} \frac{ W_{\tau1} \cdot W_{\tau1} }{W_{u1}^2}$



Machine and system curves

- Absolute head - discharge

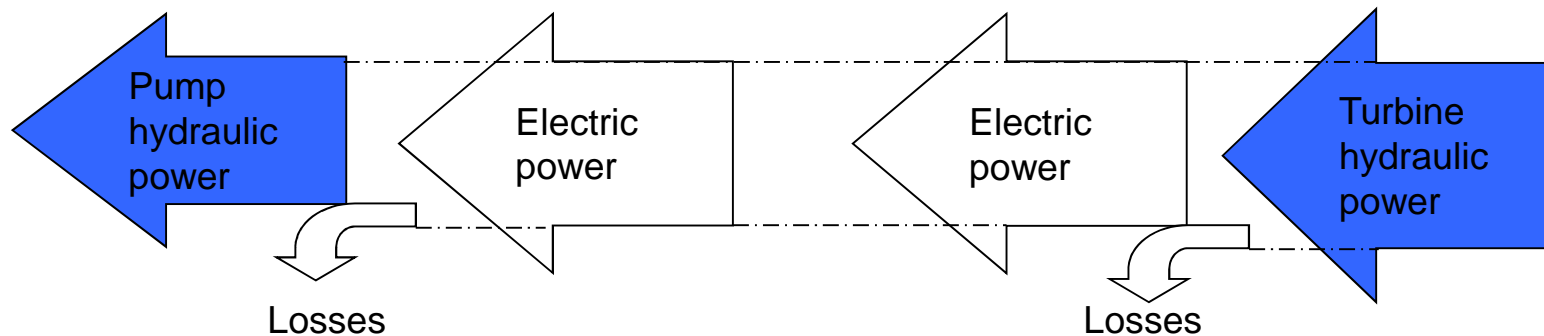
Maximum power





Machine and system curves

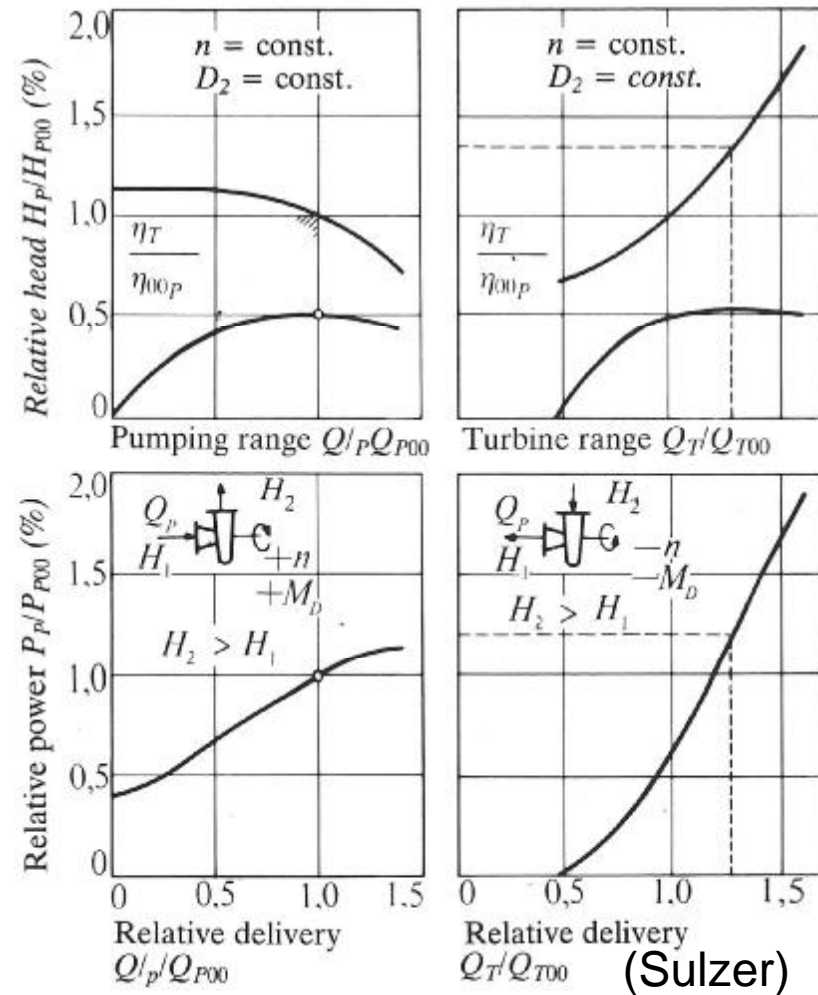
- **Maximum power in pump and turbine modes**





Machine and system curves

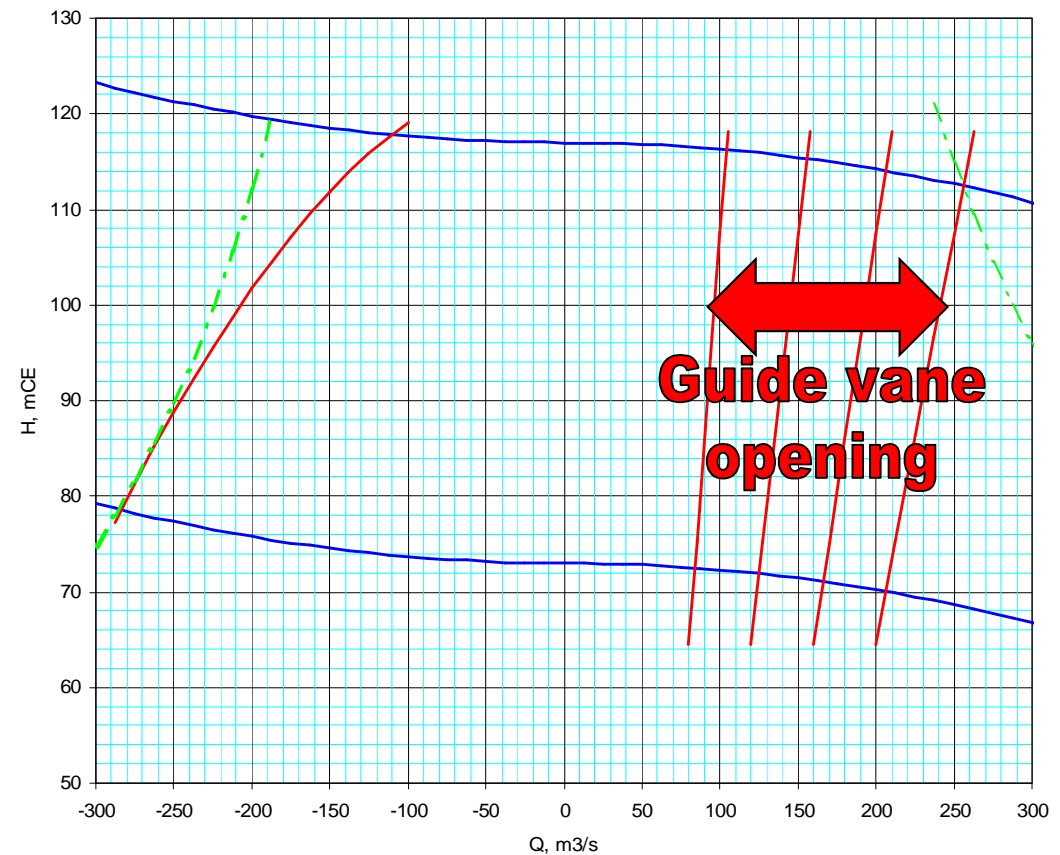
- Maximum power is at high discharge
⇒ Maximum pump power is at lowest head





Power governing in pump mode

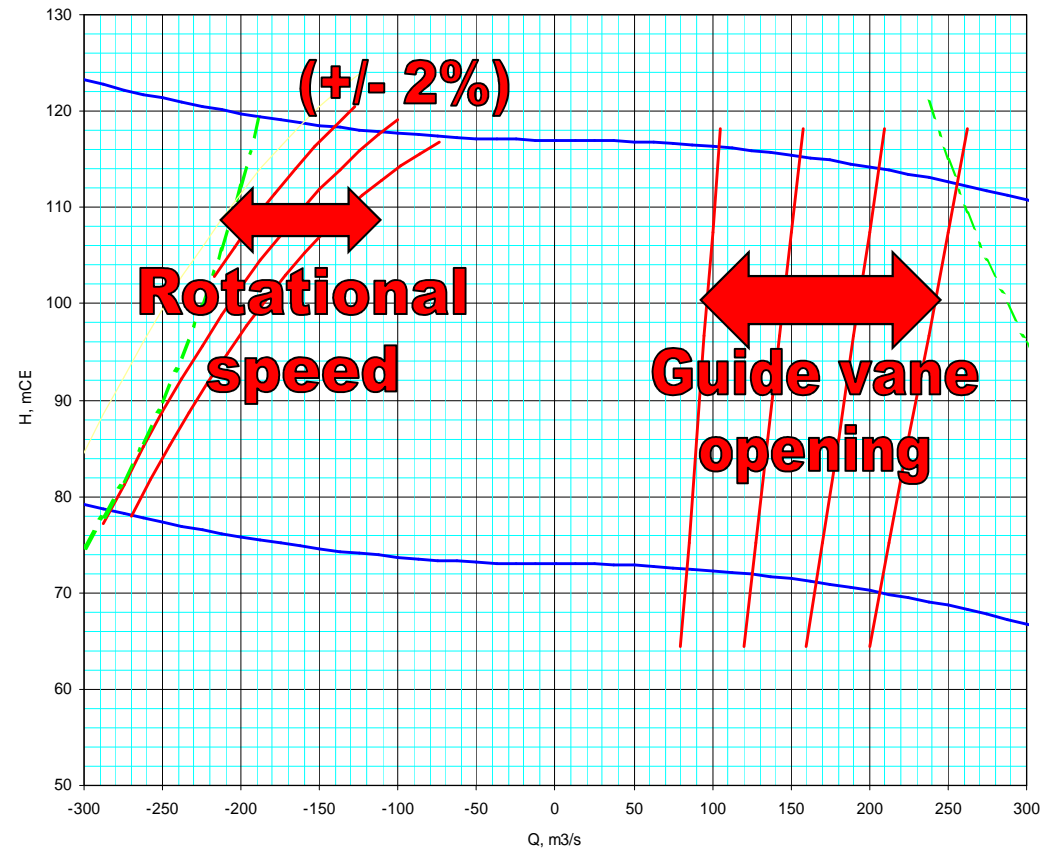
- **Constant speed: power governing in turbine mode only**





Power governing in pump mode

- Variable speed: power governing in pump mode also

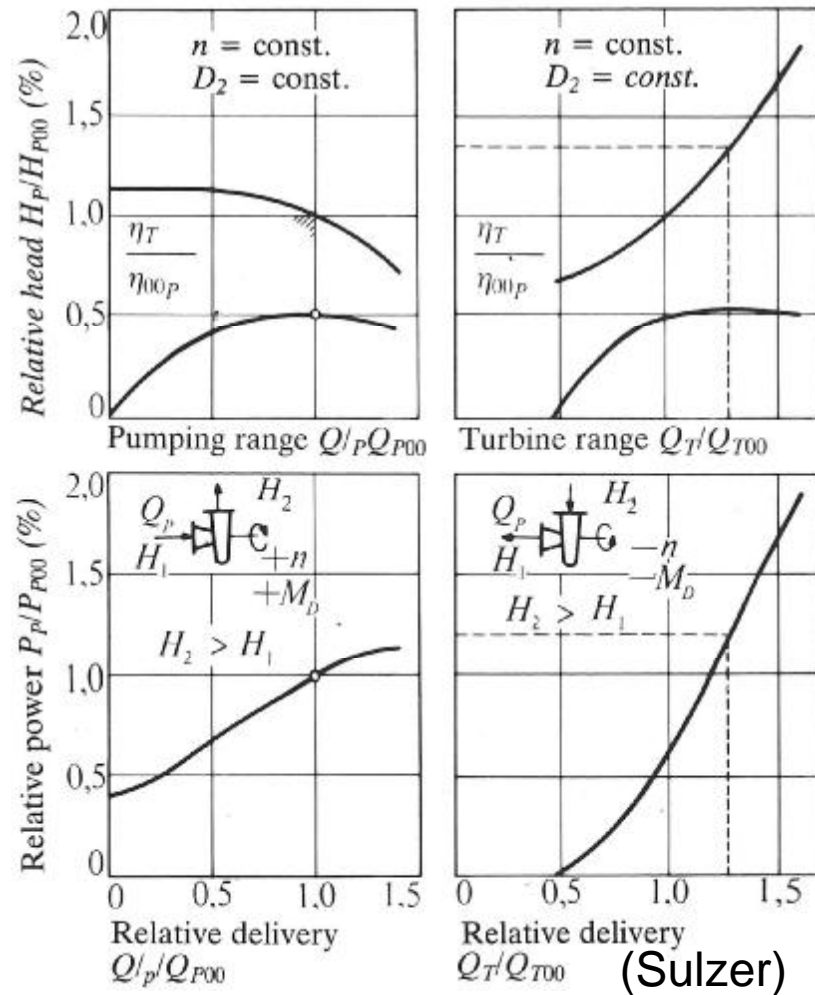




Turbine best-efficiency head

- Turbine BEP at greater head than pump BEP

⇒ Variable speed allows efficiency gain in turbine mode



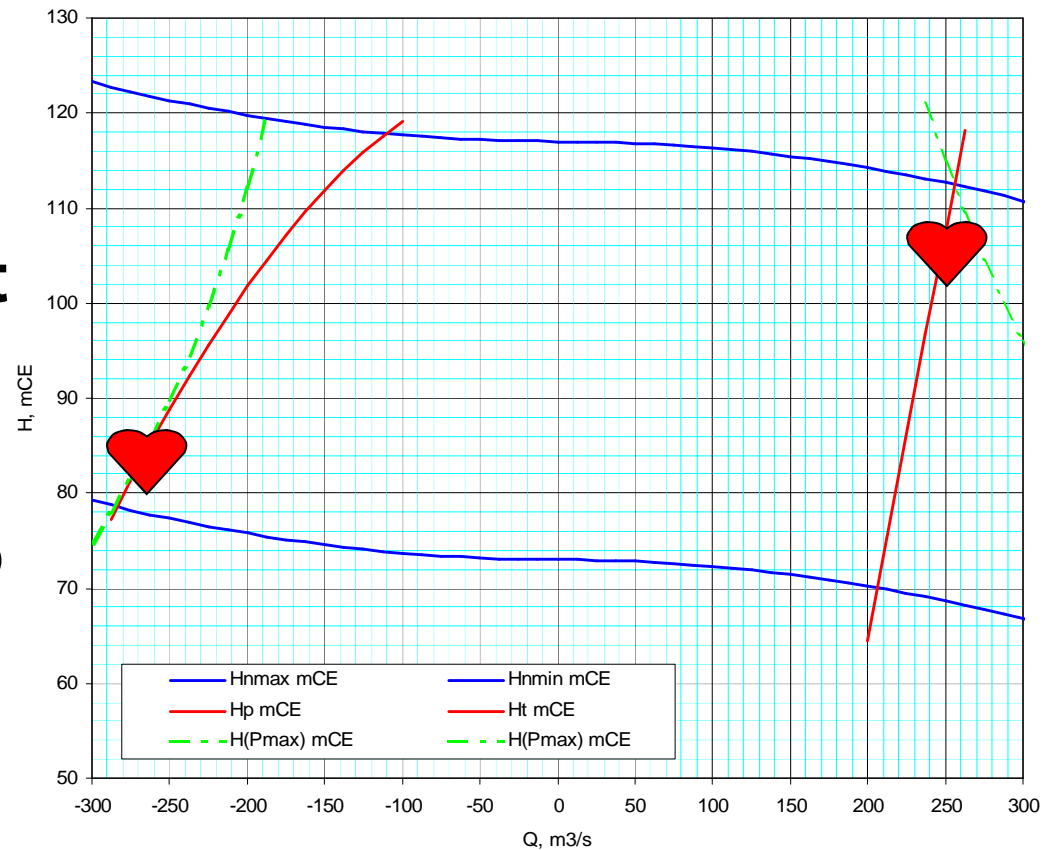


Pump mode cavitation at low head

- **BEP close to max discharge**

⇒ **Reduced efficiency at high head**

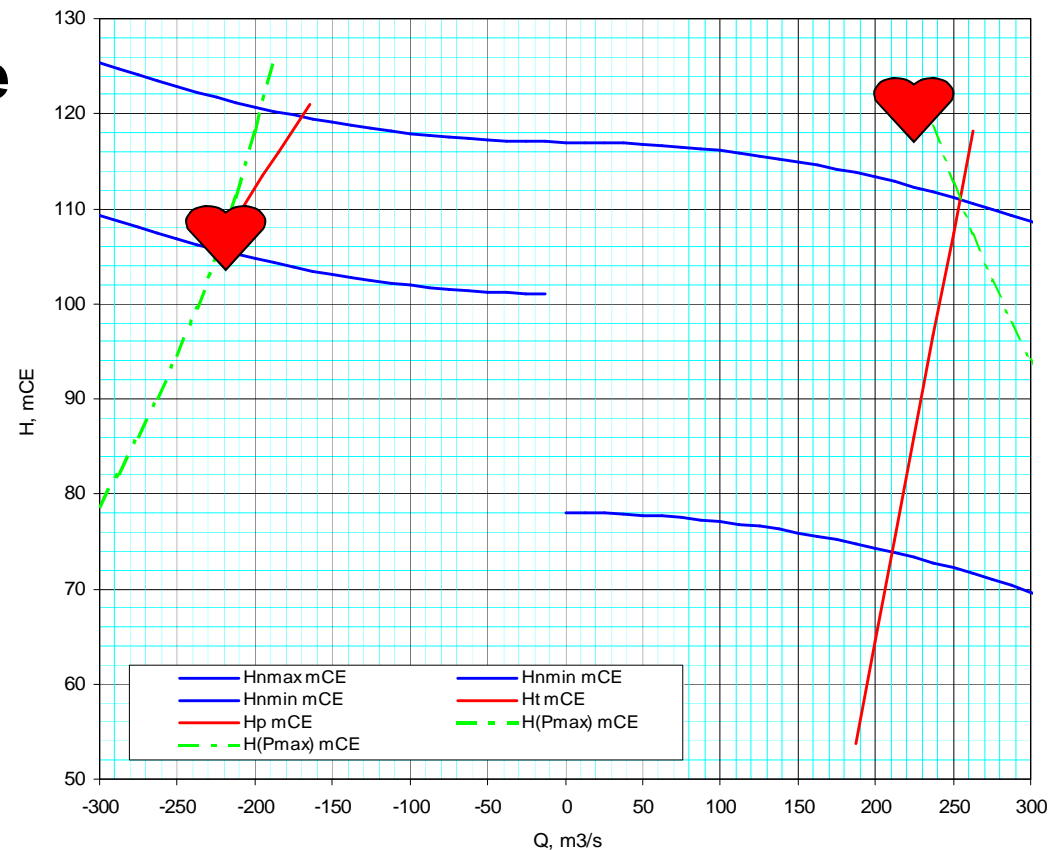
⇒ **High turbine BEP discharge**





Pump mode cavitation at low head

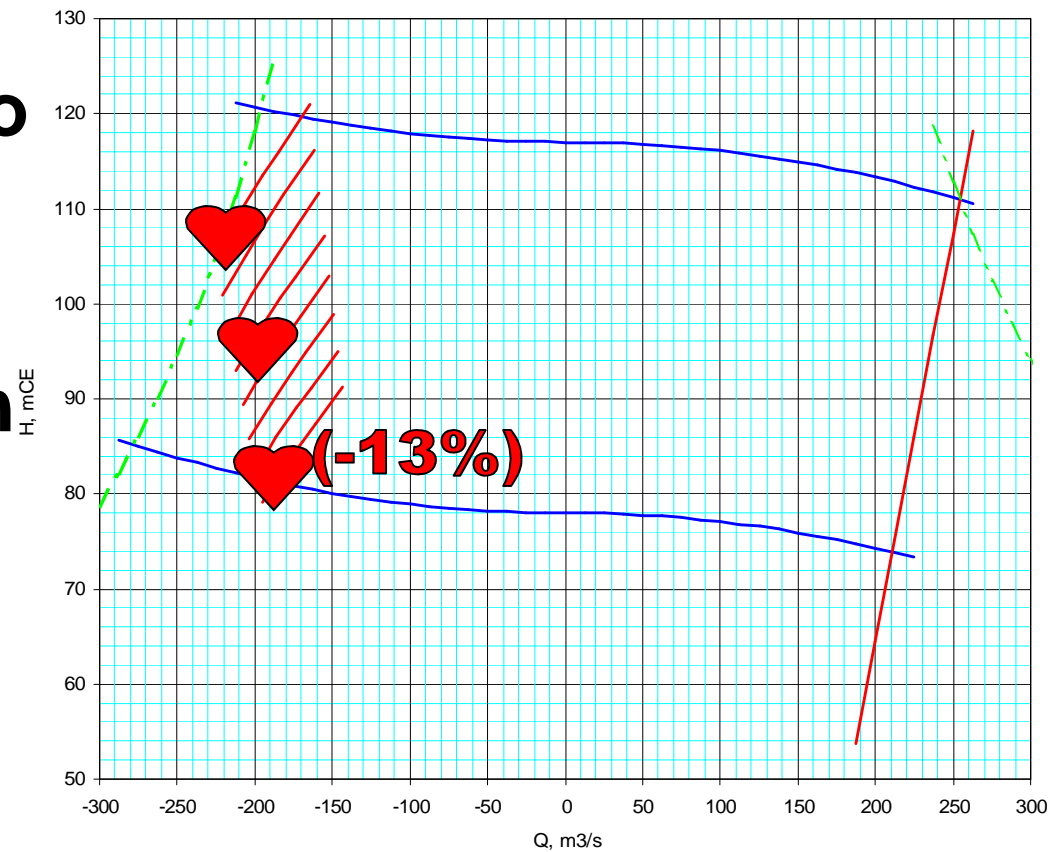
- Reduction of pump head range
⇒ Plant operation restricted
⇒ Turbine operation below BEP head





Pump mode cavitation at low head

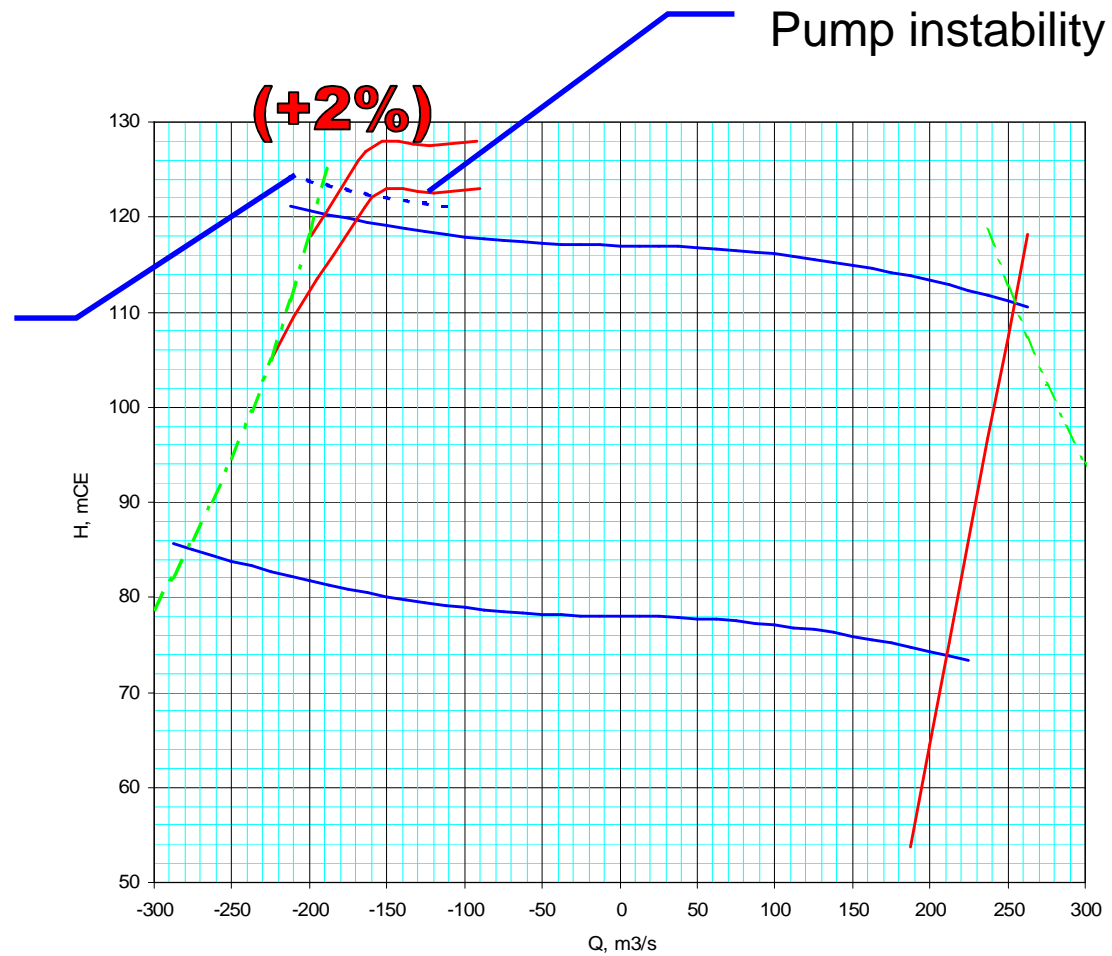
- Variable speed
 - ⇒ Little or no restriction
 - ⇒ Less submersion required at low speed





Pump start stability at high head

Transient head rise at pump start

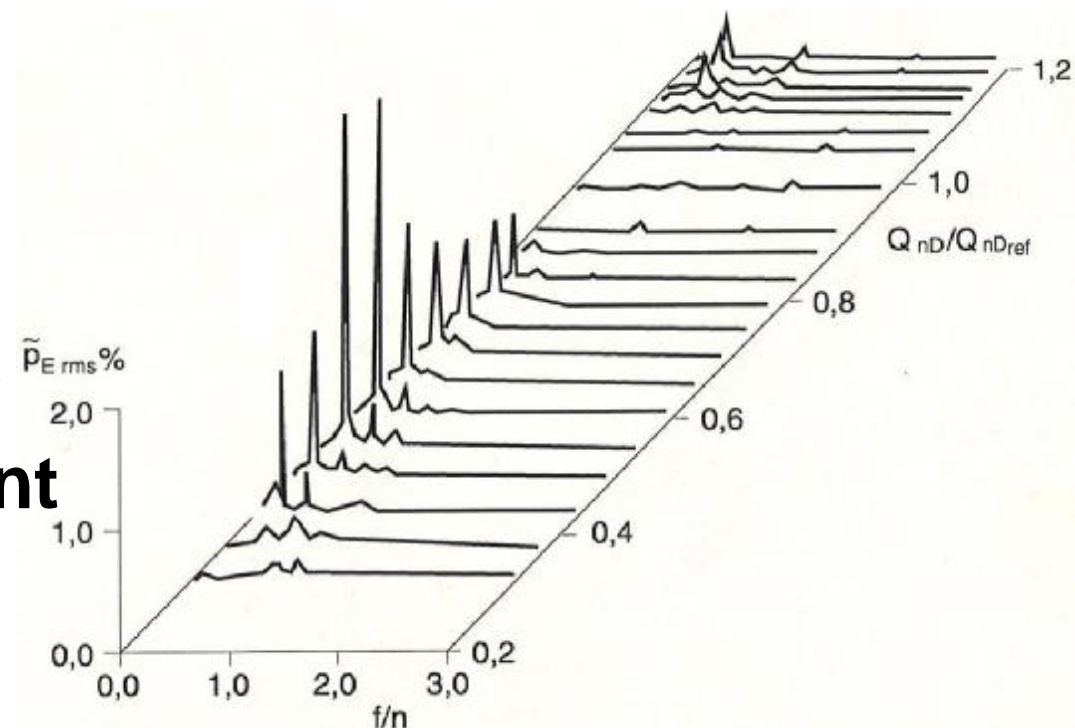




Turbine mode stability

- Hydraulic excitation frequencies are proportional to rotational speed

⇒ Variable speed may help prevent system resonance





Variable speed can:

- **Allow power governing in pump mode**
- **Improve efficiency in turbine mode**
- **Improve pump operation over large head range / reduce submersion**
- **Improve pump start stability at high head / reduce surge tank volume**
- **Help prevent hydraulic system resonance**



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Thank you for your attention